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IV Semester M.Sc. Examination, July - 2019

(CBCS-Y2K17)

MATHEMATICS

M402T : Mathematical Methods

Time : 3 Hours | Max. Marks : 70

Instructions : *(1) Answer **any five** full questions.*

*(2) **All** questions have **equal** marks.*

1. (a) Solve the Cauchy problem : **7**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; -\infty < x < \infty, t > 0$$

subject to

$u(x, 0) = f(x), \ -\infty < x < \infty$

by an appropriate fourier transform.

(b) Using Laplace transform find the solution of **7**

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = x, x > 0, t > 0$$

subject to

$u(0, t) = 0, t > 0,$

$u(x, 0) = 0, x > 0.$

2. (a) Define the Hankel transform $H\{f(r)\}$ and given the Bessel differential operator **7**

$$\Delta_\gamma \equiv \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(\frac{\gamma}{r}\right)^2,$$

find $H\{\Delta_\gamma f(r)\}$.

(b) Discuss about the limitation of fourier transforms and the need for wavelet transforms. **7**

3. Find the integral equations corresponding to : **6+8**

(a) $y''(x) + y(x) = 0$; $y(0) = 0$, $y'(0) = 0$ and

(b) $y''(x) + \lambda y(x) = 0$; $y(0) = 0$, $y(l) = 0$, (λ : constant)

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4. (a) Obtain the solution of the inhomogeneous integral equation 8

$$y(x) = G(x) + \lambda \int_a^b K(x, t) y(t) dt,$$

where $K(x, t) = \sum_{i=1}^{n} f_i(x) g_i(t)$.

(b) Using Laplace transform find the solution of 6

$$y(x) = \sin x + 2 \int_0^x \cos(x - t) y(t) dt.$$

5. For the integral 14

$$I(x) = \int_x^{\infty} e^{-t^4} dt$$

find its asymptotic expansion for both small 'x' and large 'x'.

6. (a) With the help of the Laplace method find the asymptotic expansion of 7

$$I(x) = \int_0^{\frac{\pi}{2}} e^{-x \tan t} dt \qquad \text{as } x \to \infty.$$

(b) Evaluate the following using Watson Lemma : 7

$$I(x) = \int_0^5 \frac{e^{-xt}}{1 + t^2} dt$$

7. Using a two-term perturbation method of Poincare and Lindstedt or a matched asymptotic expansion method, which ever is appropriate, find the solution of 14

$$\frac{d^2y}{dx^2} + y + \epsilon y^3 = 0 ; y(0) = a, y'(0) = 0.$$

8. Illustrate the use of the WKB method using a suitable example. 14

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